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DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

6. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Find three whole numbers the sum of any two of which is a cube.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x, y, and z=the numbers. Then $x+y=a^3$, $x+z=b^3$. and $y+z=c^3$; in which $c>b>a$. Hence $x+y+z=\dfrac{a^3+b^3+c^3}{2}$.

$$\therefore\ x=\frac{a^3+b^3+c^3}{2}-c^3;\ y=\frac{a^3+b^3+c^3}{2}-b^3;\ z=\frac{a^3+b^3+c^3}{2}-a^3.$$

In order that each of the numbers be integral, *either* each of the cubes must be even, *or* two of the cubes must be odd and one even. Also, in order that each of the numbers be positive, $a^3+b^3>c^3$.

The simple rule for finding the three numbers is as follows: Take three cubes fulfilling the above two conditions, and from half their sum subtract each cube separately; the remainders will be the three numbers required.

It is evident that many sets of numbers be obtained. We will illustrate by the following consecutive cubes: 1, (2) 8, (3) 27, (4) 64, (5) 125, (6) 216, (7) 343, (8) 512, (9) 729, (10) 1000, (11) 1331, (12) 1728, (13) 2197, (14) 2744, (15) 3375, (16) 4096, (17) 4913, (18) 5832.

The first three cubes answering the above conditions are 343, 512, and 729. ½ their sum is 792; $x=792-729=63$; $y=792-512=286$; $z=792-343=449$. The next three cubes are $(7)^3$, $(9)^3$, and $(10)^3$. The first three *even* cubes are $(12)^3$, $(14)^3$, and $(19)^3$.

9. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

It is required to find three numbers the sum of whose 4th powers is a square.

I. Solution by R. J. ADCOCK, Larchland, Warren County, Illinois.

If $u=x^2+y^2-z^2$, then $u^2=x^4+y^4+z^4+2x^2y^2-2x^2z^2-2y^2z^2$, in which if $2x^2y^2-2x^2z^2-2y^2z^2=0$, $z^2=\dfrac{x^2y^2}{x^2+y^2}$, and $u=x^2+y^2-\dfrac{x^2y^2}{x^2+y^2}$

$$=\frac{x^4+y^4+x^2y^2}{x^2+y^2};\ u^2=x^4+y^4+z^4=x^4+y^4+\frac{x^4y^4}{(x^2+y^2)^2}=\left(\frac{x^4+y^4+x^2y^2}{x^2+y^2}\right)^2.$$

$\therefore\ x^4(x^2+y^2)^2+y^4(x^2+y^2)^2+x^4y^4=(x^4+y^4+x^2y^2)^2$, which is a general equa-
for the sum of the 4th powers of three quantities=a square, when x^2+y^2=a
tion square. Making $x=3, y=4$, $x^2+y^2=5^2$, and $3^4\times5^4+4^4\times5^4+3^4\times4^4=15^4+20^4+12^4=(3^4+4^4+12^4)^2=481^2=231361$.

II. Solution by ARTEMAS MARTIN, LL. D., U. S. Coast and Geodetic Survey Office, Washington, D. C.

Let ax, ay, and az denote the numbers required. Then must $a^4x^4+a^4y^4+a^4z^4=\square$, or, rejecting the square factor a^4, we must make $x^4+y^4+z^4=\square=(x^2-y^2+z^2)^2$ say; whence $x^2z^2=x^2y^2+y^2z^2=y^2(x^2+z^2)$. Assume $x=p^2-q^2$ and $z=2pq$, and we have $xz=y(p^2+q^2)$, which gives $y=\dfrac{xz}{p^2+q^2}=\dfrac{2pq(p^2-q^2)}{p^2+q^2}$. Now take $a=p^2+q^2$ and we have $ax=p^4-q^4$, $ay=2pq(p^2-q^2)$, $az=2pq(p^2+q^2)$.

$$\therefore\ (p^4-q^4)^4+[2pq(p^2-q^2)]^4+[2pq(p^2+q^2)]^4=[(p^4-q^4)^2+12p^4q^4]^2=[(p^2+q^2)^4-4p^2q^2(p^2-q^2)^2]^2.$$

Take $p=2$, $q=1$ and we find the numbers 12, 15 and 20, the sum of whose 4th powers$=(481)^2$. These are the *smallest* numbers answering the problem, but an infinite number of other answers may be found by varying the values of p and q.

Also solved by *H. W. DRAUGHON, A. L. FOOTE, F. P. MATZ*, and *G. B. M. ZEER*.

10. Proposed by L. B. HAYWARD, Bingham, Ohio.

Find two numbers, such that each of them and also their sum and their difference when increased by unity shall all be square numbers.

I. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Maine.

The leaving out of commas leaves one in doubt whether each of the numbers *plus one*, or simply each of the numbers, must be a square. The more probable construction requires $x+1$ (1), $y+1$ (2), and $x\pm y+1$ (3) to be squares. Take $x+1=m^2+n^2$ and $y=2mn$ and the two conditions of (3) are met. But m^2+n^2 must be a square: this happens when $m=p^2-q^2$ and $n=2pq$. It now remains to make $2mn+1$, or $4pq(p^2-q^2)+1=\square=$(say) $(2pq\pm1)^2$. Reducing we have $p^2-qp=(q^2\pm1)$, from which $p=\dfrac{q\pm\sqrt{5q^2\pm4}}{2}$. $5q^2+4=\square=(sq\pm2)^2$; then $q=\pm\dfrac{4s}{s^2-5}$. We readily find $p=\dfrac{\pm2s\pm(s^2+5)}{s^2-5}$.

$s=1$, $q=\pm1$; $p=\pm1$, or ±2, $q=\pm1$, $p=\pm2$, $m=3$, and $n=4$ and $y=24$, and $x=24$. $s=2$, $q=\pm8$, $p=\pm5$ or ±13; $m=39$ and $n=80$; or $m=105$ and $n=208$; $x=7920$ or 54288; $y=6240$ or 43680. $s=3$, $q=\pm3$, $p=\pm5$ or ±2; $x=1155$ or 168; $y=960$ or 120; these last values of x and y are the least integrals which I have obtained. The reduction of $\sqrt{(5q^2-4)}=\square$ is quite interesting, but as it gives no values for x and y, different from those already obtained, I omit it.

II. Solution by A. L. FOOTE, C. E., Merrick, New York, and P. S. BERG, Apple Creek, Ohio.

Solving under the other construction, let x^2 and $2x$ be the required numbers; then the sum increased by unity is x^2+2x+1 which is a square; and their difference increased by unity is x^2-2x+1 which is also a square,